$\mathcal{E}$-Connections versus Many-Dimensionality

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Combining knowledge representation and reasoning formalisms, such as description logics (DLs), temporal logics or spatial logics, is worthwhile but difficult. It is worthwhile because usually application domains comprise various aspects of the world, thus requiring suitable combinations of available formalisms modeling each of the aspects. It is difficult because the computational behavior of the resulting hybrids is often much worse than the behavior of individual components, witness, e.g., the unpredictable behaviour of multi-dimensional modal logics.

We propose a new combination method which is robust in the computational sense and allows for certain interactions between the combined systems. The key idea of the methodology is to introduce constructs that are capable of dealing with 'link relations' which reflect the existing correspondences between objects in the domains of the combined formalisms. Typical link relations are as follows:

- 'x is in the spatial extension of y,'
- 'x belongs to the lifespan of y,'
- 'x is in the spatial extension of y at moment z,'
- object x in information system IS_1 'corresponds' to object y in information system IS_2.

The method, called $\mathcal{E}$-connection, will be defined and investigated for abstract description systems (ADS) which subsume all standard description logics, various logics of time and space, modal logics and epistemic logics. Starting from basic $\mathcal{E}$-connections, we introduce a number of additional connecting constructors—Boolean operators on links, qualified number restrictions on links, applications of link operators to object names and first-order constraints on links—and study the transfer properties of the resulting connections. For various combinations of connecting constructors we show that the $\mathcal{E}$-connection of any finite set of decidable ADSs is decidable as well. Counterexamples show that these results are optimal. The computational complexity of $\mathcal{E}$-connections is discussed.

As an illustration we show four examples: (1) $\mathcal{E}$-connections of DLs with the logic $\mathcal{MS}$ intended for reasoning about metric spaces, (2) $\mathcal{E}$-connections of DLs with the logic $\mathsf{S4}_u$ (containing $\mathsf{RCC\text{-}8}$) for qualitative representation and reasoning about topological spaces, (3) $\mathcal{E}$-connections of two (or more) DLs (which in some cases coincide with the 'distributed DLs' introduced by Borgida and Serafini) and (4) $\mathcal{E}$-connections of DLs with propositional temporal logic PTL and $\mathsf{S4}_u$.